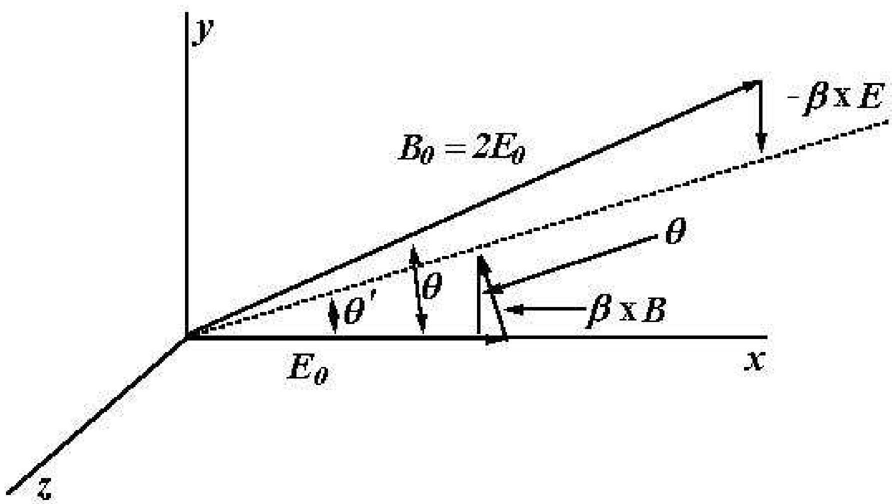
PHY 5347
Homework Set 7 Solutions – Kimel

3. 11.15
From Eq. (11.149), it is clear that we should take $\vec{\beta} \parallel \hat{z}$, so $\vec{\beta} \cdot \vec{E} = \vec{\beta} \cdot \vec{B} = 0$. Then

$$E' = \gamma(\vec{E} + \vec{\beta} \times \vec{B})$$

$$\vec{B}' = \gamma(\vec{B} - \vec{\beta} \times \vec{E})$$

The vectors in parentheses should make the same angle wrt the *x* axis θ' if they are to be parallel. This can best be seen from the figure,



From the figure

$$\vec{E}' = \gamma(E_0\hat{\imath} - \beta(2E_0)\sin\theta\hat{\imath} + \beta 2E_0\cos\theta\hat{\jmath})$$

$$\vec{B}' = \gamma(\cos\theta 2E_0\hat{\imath} + \sin\theta 2E_0\hat{\jmath} - \beta E_0\hat{\jmath})$$

Thus

$$\tan\theta' = \frac{2\beta\cos\theta}{1 - 2\beta\sin\theta} = \frac{2\sin\theta - \beta}{2\cos\theta}$$

$$(2\cos\theta)\cdot(2\beta\cos\theta) - (1 - 2\beta\sin\theta)\cdot(2\sin\theta - \beta) = 0$$

or

$$2\beta^2\sin\theta - 5\beta + 2\sin\theta = 0$$

This quadratic equation has the solution

$$\beta = \frac{1}{4\sin\theta}\left(5 - \sqrt{\left(25 - 16\sin^2\theta\right)}\right)$$

where I've chosen the solution which give $\beta = 0$ if $\theta = 0$.

If $\theta << 1$, then $\beta \to 0$, and the original fields are parallel.

If $\theta \to \pi/2$ then $\beta = \frac{1}{4}(5-3) = 1/2$. $\gamma = \frac{2}{\sqrt{3}}$

$$\vec{E}' = 0 + O(\theta - \frac{\pi}{2})\hat{\jmath}$$

$$\vec{B}' = \vec{B}' = \gamma(2E_0\hat{\jmath} - \frac{1}{2}E_0\hat{\jmath}) = \gamma E_0\frac{3}{2}\hat{\jmath}$$

So in these two limits, the fields are parallel to the *x* and *y* axes, respectively.