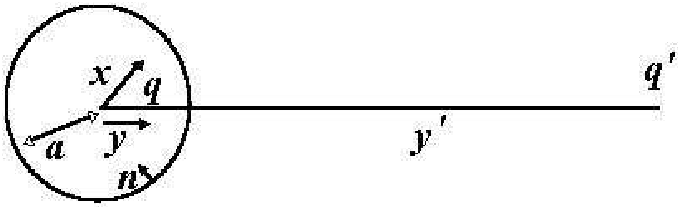
PHY 5346
HW Set 2 Solutions – Kimel

5. 2.2 The system is described by




a) Using the method of images

$$\phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{|\vec{x}-\vec{y}|} + \frac{q'}{|\vec{x}-\vec{y}'|}\right]$$

with $y' = \frac{a^2}{y}$, and $q' = -q\frac{a}{y}$

b) $\sigma = -\varepsilon_0 \frac{\partial}{\partial n}\phi|_{x=a} = +\varepsilon_0 \frac{\partial}{\partial x}\phi|_{x=a}$

$$\sigma = \varepsilon_0 \frac{1}{4\pi\varepsilon_0}\frac{\partial}{\partial x}\left[\frac{q}{(x^2+y^2-2xy\cos\gamma)^{1/2}} + \frac{q'}{(x^2+y'^2-2xy'\cos\gamma)^{1/2}}\right]$$

$$\sigma = -q\frac{1}{4\pi}\frac{a\left(1-\frac{y^2}{a^2}\right)}{(y^2+a^2-2ay\cos\gamma)^{3/2}}$$

Note

$$q_{induced} = a^2\int\sigma d\Omega = -q\frac{1}{4\pi}a^2 2\pi a\left(1-\frac{y^2}{a^2}\right)\int_{-1}^{1}\frac{dx}{(y^2+a^2-2ayx)^{3/2}}, \text{ where } x = \cos\gamma$$

$$q_{induced} = -\frac{q}{2}a(a^2-y^2)\frac{2}{a(a^2-y^2)} = -q$$

c)

$$|F| = \left|\frac{qq'}{4\pi\varepsilon_0(y'-y)^2}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q^2 ay}{(a^2-y^2)}, \text{ the force is attractive, to the right.}$$

d) If the conductor were fixed at a different potential, or equivalently if extra charge were put on the conductor, then the potential would be

$$\phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{|\vec{x}-\vec{y}|} + \frac{q'}{|\vec{x}-\vec{y}'|}\right] + V$$

and obviously the electric field in the sphere and induced charge on the inside of the sphere would remain unchanged.