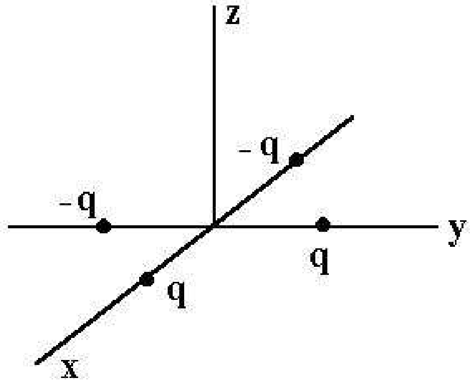
PHY 5346
HW Set 6 Solutions – Kimel

3. 4.1



$$q_{lm} = \int r^l Y_l^{m*}(\theta,\phi)\rho(\vec{x})d^3x = \sum_i q_i r_i^l Y_l^{m*}(\theta_i,\phi_i)$$

Using

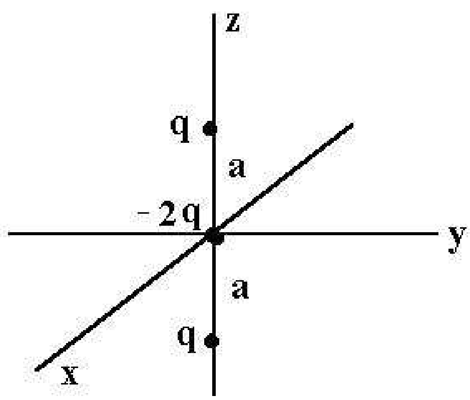
$$Y_l^{m*}(\theta,\phi) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(x)e^{--m\phi} = N_l^m P_l^m(x)e^{--m\phi}$$

From the figure we get

$$q_{lm} = a^l N_l^m P_l^m(0) q[(1-(-1)^m)(1-i^m)] = 0, \text{ for } m \text{ even, so } m = 2n+1,\ n = 0,1,2,\ldots$$

$$q_{lm} = 2qa^l N_l^m P_l^m(0)[(1-(-1)^n i)]$$

b) The figure for this system is



Since the sum of the charges equals zero, $l \geq 1$.

$$q_{lm} = qa^l[Y_l^{m*}(x=1,\phi) + Y_l^{m*}(x=-1,\phi)] = qa^l N_l^m[P_l^m(1) + P_l^m(-1)]$$

From the Rodrigues formula for $P_l^m(x)$, we see $P_l^m(\pm 1) = 0$, for $m \neq 0$. So

$$q_{lm} = qa^l N_l^0[1+(-1)^l]P_l(1)$$

Thus l is even, but $l \neq 0$

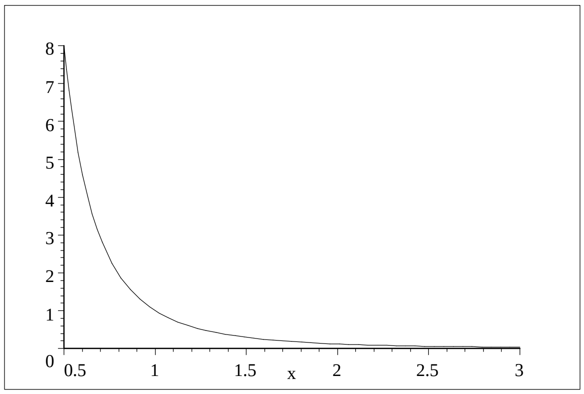
$$q_{lm} = 2qa^l N_l^0$$

c) Using the fact that $N_l^0 = \sqrt{\frac{2l+1}{4\pi}}$ and $Y_l^0 = \sqrt{\frac{2l+1}{4\pi}} P_l$

$$\Phi(\vec{x}) = \sum_{l=2}^{\infty} (2qa^l) \frac{P_l(x)}{r^{l+1}} \approx \frac{2qa^2}{r^3} P_2(x = 0 \text{ on x-y plane})$$

$$\Phi(\vec{x}) = -\frac{qa^2}{r^3}$$

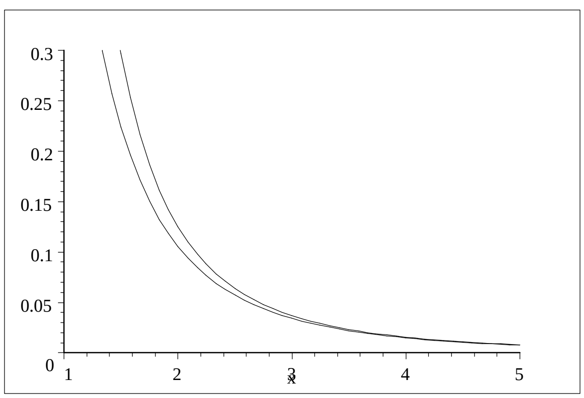
Let us plot $\Phi(\vec{x})/(-q/a)$, ie, $\frac{1}{(\frac{r}{a})^3} = \frac{1}{x^3}$



The exact answer on the x-y plane is

$$\Phi(\vec{x}) = \frac{-q}{a}\left[\frac{2}{x} - \frac{2}{x\sqrt{1+\frac{1}{x^2}}} \right] = \frac{-q}{a}\left(\left(\frac{1}{x}\right)^3 - \frac{3}{4}\left(\frac{1}{x}\right)^5 + \frac{5}{8}\left(\frac{1}{x}\right)^7 - \frac{35}{64}\left(\frac{1}{x}\right)^9 + \ldots \right)$$

So let's plot $\frac{1}{x^3}, \frac{2}{x} - \frac{2}{x\sqrt{1+\frac{1}{x^2}}}$



where the smaller is the exact answer.