PHY 5346
HW Set 7 Solutions – Kimel

2. 4.2 We want to show that we can obtain the potential and potential energy of an elementary diplole:

$$\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\bullet\vec{x}}{r^3}$$

$$W = -\vec{p}\bullet\vec{E}(0)$$

from the general formulas

$$\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{x}')d^3x'}{|\vec{x}-\vec{x}'|}$$

$$W = \int\rho(\vec{x})\Phi(\vec{x})d^3x$$

using the effective charge density

$$\rho_{eff} = -\vec{p}\bullet\vec{\nabla}\delta(\vec{x})$$

where I've chosen the origin to be at $\vec{x}_0$.

$$\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\int\frac{-\vec{p}\bullet\vec{\nabla}'\delta(\vec{x}')d^3x'}{|\vec{x}-\vec{x}'|} = -\frac{1}{4\pi\varepsilon_0}\vec{p}\bullet\int\vec{\nabla}\left(\frac{1}{|\vec{x}-\vec{x}'|}\right)\delta(\vec{x}')d^3x'$$

$$\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\bullet\vec{x}}{r^3}$$

Similarly,

$$W = \int\rho(\vec{x})\Phi(\vec{x})d^3x = -\int\vec{p}\bullet\vec{\nabla}\delta(\vec{x})\Phi(\vec{x})d^3x = \vec{p}\bullet\int\delta(\vec{x})\vec{\nabla}\Phi(\vec{x})d^3x = -\vec{p}\bullet\vec{E}(0)$$