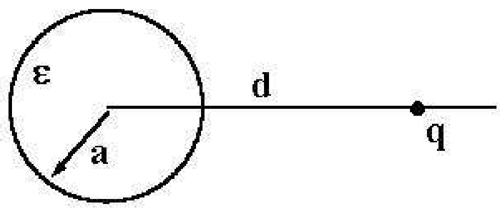
PHY 5346
HW Set 7 Solutions – Kimel

4. 4.9 a) The system is described by



Since there is azimuthal symmetry, choosing the z-axis through q,

$$\Phi_{out} = \frac{1}{4\pi\varepsilon_0}\left(\sum_l B_l r^{-l-1}P_l + \frac{q}{|\vec{x}-\vec{x}'|}\right)$$

$$\Phi_{out} = \frac{1}{4\pi\varepsilon_0}\left(\sum_l B_l r^{-l-1}P_l + \frac{q}{r_>}\sum_l\left(\frac{r_<}{r_>}\right)^l P_l\right)$$

$$\Phi_{in} = \frac{1}{4\pi\varepsilon_0}\left(\sum_l A_l r^l P_l + \frac{q}{r_>}\sum_l\left(\frac{r_<}{r_>}\right)^l P_l\right)$$

Boundary conditions: At the surface, $r' = d = r_>,\ r = a = r_<$.

1) $\Phi_{out} = \Phi_{in}|_{r=a}$, or

$$B_l = A_l a^{2l+1}$$

2) $\varepsilon\frac{\partial}{\partial r}\Phi_{in} = \frac{\partial}{\partial r}\Phi_{out}|_{r=a}$, or letting $k = \frac{\varepsilon}{\varepsilon_0}$

$$k\left[\sum_l lA_l a^{l-1}P_l + \frac{q}{d}l\left(\frac{a^{l-1}}{d^l}\right)P_l\right] = \left[\sum_l -(l+1)B_l a^{-l-2}P_l + \frac{q}{d}l\left(\frac{a^{l-1}}{d^l}\right)P_l\right]$$

$$= \left[\sum_l -(l+1)A_l a^{l-1}P_l + \frac{q}{d}l\left(\frac{a^{l-1}}{d^l}\right)P_l\right]$$

or

$$A_l = \frac{a(1-k)l}{[(1+k)l+1]d^{l+1}}$$

$$B_l = \frac{a(1-k)la^{2l+1}}{[(1+k)l+1]d^{l+1}}$$

Remember that $P_l = \sqrt{\frac{4\pi}{2+1}}\ Y_l^0$, and substitute the above coefficients into the expansion to get the answer requested by the problem.