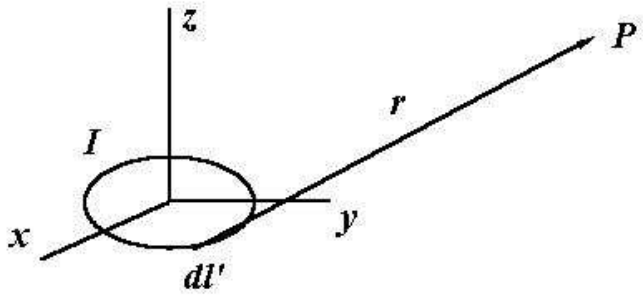
PHY 5346
Homework Set 10 Solutions – Kimel

1. 5.1 The system is described by



We want to show

$$\phi_m = -\frac{\mu_0 I}{4\pi}\Omega$$

Suppose the observation point is moved by a displacement $\delta\vec{x}$, or equivlently that the loop is displaced by $-\delta\vec{x}$.

If we are to have $\vec{B} = -\vec{\nabla}\phi_m$, then

$$\delta\phi_m = -\delta\vec{x}\cdot\vec{B}$$

Using the law of Biot and Savart,

$$\delta\phi_m = -\frac{\mu_0 I}{4\pi}\oint \delta\vec{x}\cdot\frac{\left(d\vec{l}'\times\vec{r}\right)}{r^3} = -\frac{\mu_0 I}{4\pi}\oint \vec{r}\cdot\frac{\left(\delta\vec{x}\times d\vec{l}'\right)}{r^3} = -\frac{\mu_0 I}{4\pi}\oint \hat{r}\cdot\frac{\left(\delta\vec{x}\times d\vec{l}'\right)}{r^2}$$

$$\delta\phi_m = -\frac{\mu_0 I}{4\pi}\oint \frac{\hat{r}\cdot\delta(dA)}{r^2} = -\frac{\mu_0 I}{4\pi}\delta\Omega$$

Or,

$$\phi_m = -\frac{\mu_0 I}{4\pi}\Omega$$