

PHY 5346  
Homework Set 9 Solutions – Kimel

1. 5.10

a) From Eq. (5.35)

$$A_\phi(r, \theta) = \frac{\mu_0}{4\pi} \frac{I}{a} \int \frac{r'^2 dr' d\Omega' \sin\theta' \cos\phi' \delta(\cos\theta') \delta(r' - a)}{|\vec{x} - \vec{x}'|}$$

Using the expansion of  $1/|\vec{x} - \vec{x}'|$  given by Eq. (3.149),

$$\frac{1}{|\vec{x} - \vec{x}'|} = \frac{4}{\pi} \int_0^\infty dk \cos[k(z - z')] \left\{ \frac{1}{2} I_0(k\rho_<) K_0(k\rho_>) + \sum_{m=1}^\infty \cos[m(\phi - \phi')] I_m(k\rho_<) K_m(k\rho_>) \right\}$$

We orient the coordinate system so  $\phi = 0$ , and because of the  $\cos\phi'$  factor,  $m = 1$ . Thus,

$$A_\phi(r, \theta) = \frac{\mu_0}{4\pi} \frac{I}{a} \frac{4\pi}{\pi} \int_0^\infty dk \int r'^2 dr' d\cos\theta' \sin\theta' \delta(\cos\theta') \delta(r' - a) \cos(kz) I_1(k\rho_<) K_1(k\rho_>)$$

$$A_\phi(r, \theta) = \frac{\mu_0}{\pi} a I \int_0^\infty dk \cos(kz) I_1(k\rho_<) K_1(k\rho_>)$$

where  $\rho_<(\rho_>)$  is the smaller (larger) of  $a$  and  $\rho$ .

b) From problem 3.16 b),

$$\frac{1}{|\vec{x} - \vec{x}'|} = \sum_{m=-\infty}^\infty \int_0^\infty dk e^{im(\phi - \phi')} J_m(k\rho) J_m(k\rho') e^{-k|z|}$$

Note  $z' = 0$ , and  $\phi = 0$ , so

$$A_\phi = \frac{\mu_0 I a}{2} \int_0^\infty dk e^{-k|z|} J_1(k\rho) J_1(ka)$$