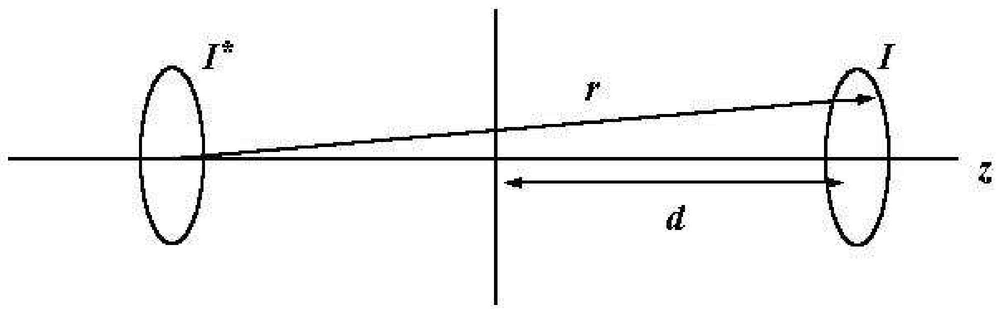
PHY 5346
Homework Set 9 Solutions – Kimel

2. 5.18

a) From the results of Problem 5.17, we can replace the problem stated by the system



where I* is equidistant from the interface and is equal to $I^* = \frac{\mu_r - 1}{\mu_r + 1} I$. The radius of each current loop is a. Now from Eq. (5.7)

$$\vec{F}(on\ I) = I \int \vec{dl} \times B(\vec{r})$$

$$\vec{dl} \times \vec{B} = \vec{dl} \times \vec{B}_r + \vec{dl} \times \vec{B}_\theta = dlB_r\left(-\hat{\theta}\right) + dlB_\theta \hat{r}$$

By symmetry, only the z – component survives, so, from the figure

$$\left(\vec{dl} \times \vec{B}\right) \cdot \hat{z} = dlB_r\left(\frac{a}{\sqrt{4d^2 + a^2}}\right) + dlB_\theta\left(\frac{2d}{\sqrt{4d^2 + a^2}}\right)$$

So

$$F_z = \frac{2\pi a I}{\sqrt{4d^2 + a^2}}\left[aB_r + 2dB_\theta\right]$$

with B_r and B_θ given by Eqs. (5.48) and (5.49) and $\cos\theta = \frac{2d}{\sqrt{4d^2 + a^2}}$, $r = \sqrt{4d^2 + a^2}$, and $I \to I^*$.

c) To determine the limiting term, simply let $r \to 2d$ and take the lowest non-vanishing term in the expansion of the magnetic flux density.

$$F_z = \frac{\pi a I}{d}\left[aB_r + 2dB_\theta\right]$$

$$F_z = \frac{\pi a I}{d}\left[a\left(\frac{\mu_0 I^* a}{4d}\frac{a}{(2d)^2}\right) + 2d\left(-\frac{\mu_0 I^* a^2}{4}\left(\frac{1}{(2d)^3}\right)\right)\left(-\frac{a}{2d}\right)\right]$$

$$F_z \to -\frac{3\pi\mu_0}{32}\frac{a^4 I \times I^*}{d^4}$$

The minus sign shows the force is attractive if I and I^* are in the same direction. This same result can be gotten more directly, using

$$F_z = \nabla_z(mB_z)$$

with $m = \pi a^2 I,$ and (from Eq. (5.64))

$$B_z = \frac{\mu_0}{4\pi}\left(\frac{2m^*}{z^3}\right)$$

with $m^* = \pi a^2 I^*,$ and $z = 2d$

$$F_z = \frac{\mu_0}{4\pi} 2\pi a^2 I^* \pi a^2 I \left(-\frac{3}{(2d)^4}\right) = -\frac{3\pi\mu_0}{32}\frac{a^4 I \times I^*}{d^4}$$

with agrees with out previous result.