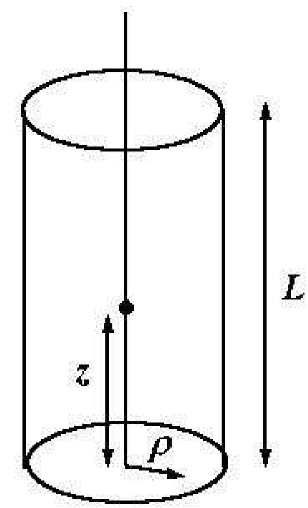
PHY 5346
Homework Set 9 Solutions – Kimel

3. 5.19 The system is described by



The effective volume magnetic charge density is zero, since $\vec{M}$ is constant within the cylinder. The effective surface charge density ($\hat{n} \cdot \vec{M}$ from Eq. (5.99)) is M_0, on the top surface and $-M_0$ on the bottom surface. From the bottom surface the potential is (for $z > 0$)

$$\Phi_b = \frac{1}{4\pi}(-M_0)2\pi \int_0^a \frac{\rho d\rho}{(\rho^2 + z^2)^{1/2}} = -\frac{M_0}{2}\left(\sqrt{(a^2 + z^2)} - z\right)$$

By symmetry, the potential from the top surface is (on the inside)

$$\Phi_t = \frac{M_0}{2}\left(\sqrt{\left(a^2 + (L-z)^2\right)} - (L-z)\right)$$

The total magnetic potential is

$$\Phi = \Phi_b + \Phi_t = -\frac{M_0}{2}\left(\sqrt{(a^2 + z^2)} - z\right) + \frac{M_0}{2}\left(\sqrt{\left(a^2 + (L-z)^2\right)} - (L-z)\right)$$

So, on the inside of the cylinder,

$$H_z = -\frac{\partial}{\partial z}\left(-\frac{M_0}{2}\left(\sqrt{(a^2 + z^2)} - z\right) + \frac{M_0}{2}\left(\sqrt{\left(a^2 + (L-z)^2\right)} - (L-z)\right)\right)$$

$$H_z = -\frac{M_0}{2}\left[2 - \frac{z}{\sqrt{(a^2 + z^2)}} - \frac{L-z}{\sqrt{\left(a^2 + (L-z)^2\right)}}\right]$$

while above the cylinder,

$$H_z = -\frac{M_0}{2}\left[-\frac{z}{\sqrt{(a^2+z^2)}} + \frac{z-L}{\sqrt{\left(a^2+(L-z)^2\right)}}\right]$$

with a similar expression below the cylinder.

$$\vec{B} = \mu_0\left(\vec{H}+\vec{M}\right)$$

Thus inside the cylinder,

$$B_z = \mu_0\left[-\frac{M_0}{2}\left[2-\frac{z}{\sqrt{(a^2+z^2)}} - \frac{L-z}{\sqrt{\left(a^2+(L-z)^2\right)}}\right] + M_0\right]$$

$$B_z = \frac{\mu_0 M_0}{2}\left(\frac{z}{\sqrt{(a^2+z^2)}} + \frac{L-z}{\sqrt{\left(a^2+(L-z)^2\right)}}\right)$$

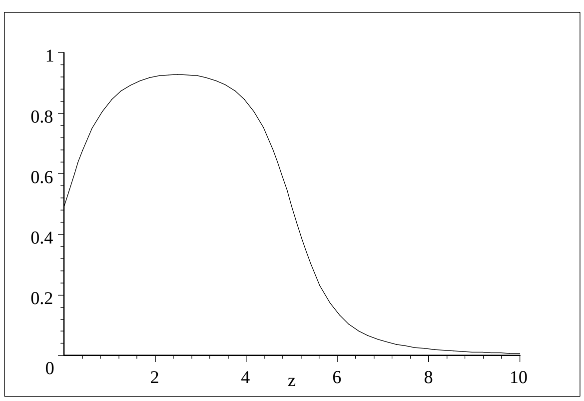
while above the cylinder,

$$B_z = \frac{\mu_0 M_0}{2}\left[\frac{z}{\sqrt{(a^2+z^2)}} - \frac{z-L}{\sqrt{\left(a^2+(L-z)^2\right)}}\right]$$

First we plot B_z in units of a for $L = 5a$

$$g(z) = \begin{cases} \frac{1}{2}\left(\frac{z}{\sqrt{(1+z^2)}} + \frac{5-z}{\sqrt{(1+(5-z)^2)}}\right) & \text{if} \quad z < 5 \\ \frac{1}{2}\left(\frac{z}{\sqrt{(1+z^2)}} - \frac{z-5}{\sqrt{(1+(5-z)^2)}}\right) & \text{if} \quad 5 < z \end{cases}$$

$g(z)$



And similarly, H_z in units of a for $L = 5a$.

$$f(z) = \begin{cases} -\frac{1}{2}\left(2 - \frac{z}{\sqrt{(1+z^2)}} - \frac{5-z}{\sqrt{(1+(5-z)^2)}}\right) & \text{if} \quad z < 5 \\ -\frac{1}{2}\left(-\frac{z}{\sqrt{(1+z^2)}} + \frac{z-5}{\sqrt{(1+(5-z)^2)}}\right) & \text{if} \quad 5 < z \end{cases}$$

$f(z)$