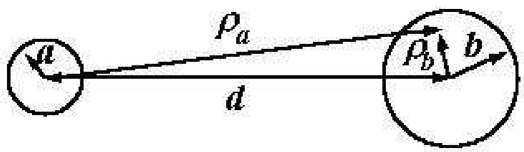
PHY 5346
Homework Set 11 Solutions – Kimel

3. 5.29 The system is described by



This problem is very much like 5.26, except the wires are superconducting. We know from section 5.13 that the magnetic field within a superconductor is zero. We will be using

$$W = \frac{1}{2}\int \vec{J}\cdot\vec{A}d^3x = \frac{l}{2}\int[J_aAda_a + J_bAda_b]$$

Using the same arguments as applied in problem 5.26,

$$A_z = \begin{cases} -\frac{\mu I}{2\pi}\left(\ln\frac{\rho}{R} + C\right) = -\frac{\mu I}{4\pi}\left(\ln\frac{\rho^2}{R^2} + 0\right) & \text{on the outside} \\ 0, & \text{on the inside} \end{cases}$$

Thus if we consider the second term $\frac{l}{2}\int J_bAda_b$,

$$\frac{l}{2}\int J_bAda_b = \frac{l}{2}\frac{I}{\pi b^2}\int[A_{out}(\rho_a) + A_{in}(\rho_{b)})]\rho_b d\rho_b d\phi$$

$$\simeq \frac{l}{2}\frac{I}{\pi b^2}\frac{\mu I}{4\pi}2\pi\int_0^b \ln\frac{d^2}{a^2}\rho_b d\rho_b = \frac{l}{2}\left(\frac{\mu}{4\pi}\right)\left(2\ln\frac{d}{a}\right)I^2$$

The first term $\frac{l}{2}\int J_aAda_a$ is equal to

$$\frac{l}{2}\int J_aAda_a = \frac{l}{2}\left(\frac{\mu}{4\pi}\right)\left(2\ln\frac{d}{b}\right)I^2$$

Thus

$$W = \frac{l}{2}\left(\frac{\mu}{4\pi}\right)\left(2\ln\frac{d^2}{ab}\right)I^2 = \frac{l}{2}\frac{L}{l}I^2$$

so

$$\frac{L}{l} = \left(\frac{\mu}{4\pi}\right)\left(2\ln\frac{d^2}{ab}\right)$$

Now using the methods of problem 1.6, assuming the left wire has charge Q, and the right wire charge $-Q$, we find

$$\phi_{12} = \int_b^{d-a} Edr = \frac{\frac{Q}{l}}{2\pi\varepsilon}\int_b^{d-a}\left(\frac{1}{r} + \frac{1}{d-r}\right)dr \simeq \frac{\frac{Q}{l}}{2\pi\varepsilon}\ln\frac{d^2}{ab}$$

$$\frac{C}{l} = \frac{\frac{Q}{l}}{\phi_{12}} = \frac{2\pi\varepsilon}{\ln\frac{d^2}{ab}}$$

Thus

$$\frac{L}{l} \times \frac{C}{l} = \left(\frac{\mu}{4\pi}\right)\left(2\ln\frac{d^2}{ab}\right) \times \frac{2\pi\varepsilon}{\ln\frac{d^2}{ab}} = \mu\varepsilon$$