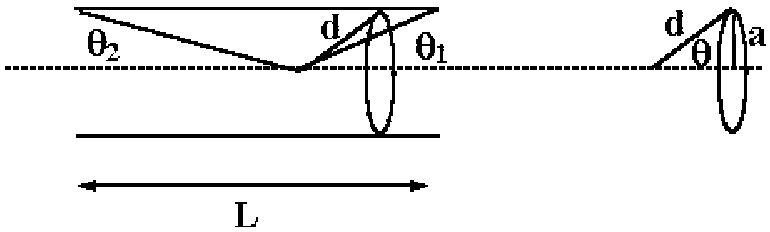
PHY 5346
HW Set 10 Solutions – Kimel

1. 5.3 The system is described by



The law of Biot and Savart says

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$$

From the figure, for one loop

$$B_z = \frac{\mu_0}{4\pi}\frac{I2\pi a\sin\theta}{d^2} = \frac{\mu_0}{4\pi}\frac{I2\pi\sin^3\theta}{a}$$

As $NL\to\infty$, $dN = Ndz$, but $\frac{d\theta}{dz} = \frac{\sin\theta}{d}$, $d = \frac{a}{\sin\theta}$, so $dN = N\frac{ad\theta}{\sin^2\theta}$

$$B_{ztot} = \int B_z dN = \frac{\mu_0}{4\pi}I2\pi N\int_{\theta_2}^{\pi-\theta_1}\sin\theta d\theta = \frac{\mu_0 IN}{2}\left[\cos\theta_2 - \cos\left(\pi-\theta_1\right)\right]$$

$$B_{ztot} = \frac{\mu_0 IN}{2}\left[\cos\theta_2 + \cos\theta_1\right]$$