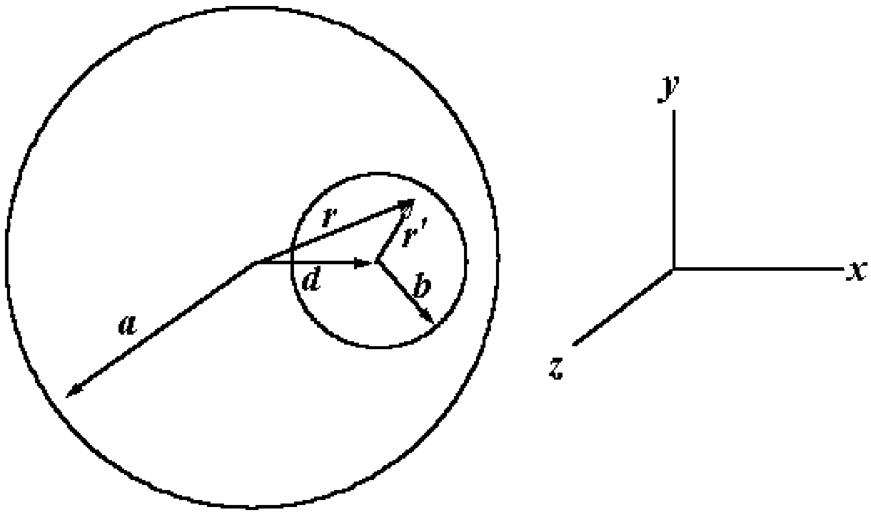
PHY 5346
HW Set 10 Solutions – Kimel

2. 5.6 We may choose the coordinate system so the currents and hole are aligned as



Here, I'm taking the *z* axis as out of the paper. Then, applying the superposition principle, we can replace this system by one where a current $\vec{J}$ fills the whole wire and is in the z direction, while an opposite current $\vec{J}' = -\vec{J}$ flows in a wire the size of the hole and is located where the hole previously was.

From Ampere's law we can work out the magnitude of the magnetic flux density

$$\int \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{a} = \mu_0 J \pi r^2 = B 2\pi r \rightarrow B = \frac{\mu_0 J r}{2}$$

Similarly

$$B' = \frac{\mu_0 J r'}{2}$$

Putting in the directions

$$\vec{B} = \frac{\mu_0 J \hat{z} \times \vec{r}}{2}$$

and

$$\vec{B}' = \frac{\mu_0 J \left(-\hat{z}\right) \times \vec{r}'}{2}$$

$$\vec{B}_{tot} = \vec{B} + \vec{B}' = \frac{\mu_0 J \hat{z} \times \left(\vec{r} - \vec{r}'\right)}{2}$$

However, from the figure, $\vec{r} = \vec{d} + \vec{r}'$, so

$$\vec{B}_{tot} = \frac{\mu_0 J \hat{z} \times \vec{d}}{2} = \frac{\mu_0 J d}{2} \hat{y}$$

Thus we conclude the magnetic flux density in the hole is a constant, $B_{tot} = \frac{\mu_0 J d}{2}$, and it is directed in the y direction.