PHY 5346
Homework Set 10 Solutions – Kimel

3. 5.8 Using the same arguments that lead to Eq. (5.35), we can write

$$A_\phi = \frac{\mu_0}{4\pi}\int \frac{d^3x' \cos\phi' J_\phi(r',\theta')}{|\vec{x}-\vec{x}'|}$$

Choose $\vec{x}$ in the $x - z$ plane. Then we use the expansion

$$\frac{1}{|\vec{x}-\vec{x}'|} = \sum_{l,m} \frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}} Y_l^{m*}(\theta',\phi')Y_l^m(\theta,0)$$

The $\cos\phi'$ factor leads to only an $m = 1$ contribution in the expansion. Using

$$Y_l^m(\theta,0) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)$$

and $\frac{(l-1)!}{(l+1)!} = \frac{1}{l(l+1)}$, we have on the **inside**

$$A_\phi = \frac{\mu_0}{4\pi}\sum_l \frac{1}{l(l+1)} r^l P_l^1(\cos\theta)\int d^3x' \frac{P_l^1(\cos\theta')J_\phi(r',\theta')}{r'^{l+1}}$$

which can be written

$$A_\phi = -\frac{\mu_0}{4\pi}\sum_l m_l r^l P_l^1(\cos\theta)$$

with

$$m_l = -\frac{1}{l(l+1)}\int d^3x' \frac{P_l^1(\cos\theta')J_\phi(r',\theta')}{r'^{l+1}}$$

A similar expression can be written on the **outside** by redefining $r_<$ and $r_>$.