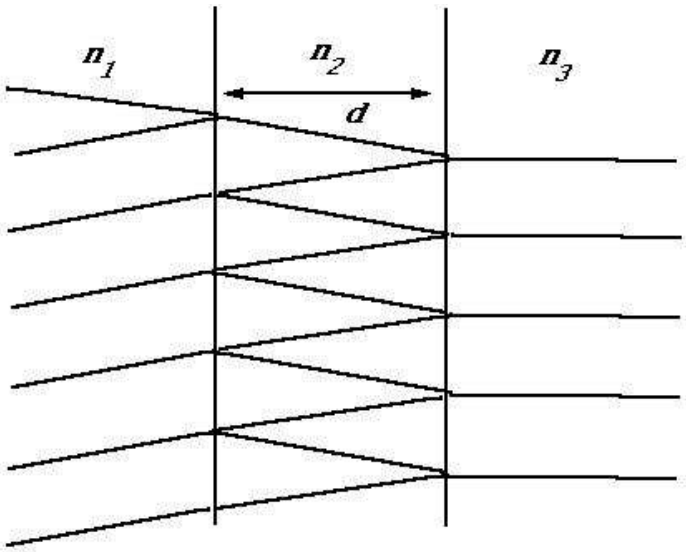
PHY 5346
Homework Set 13 Solutions – Kimel

1. 7.2

a) The figure describes the muliple internal reflections which interfere to give the overall reflection and refraction:



For the *ij* interface I shall use the notation

$$r_{ij} = \frac{E_0'}{E_0} = \frac{2n_i}{n_i + n_j}$$

$$R_{ij} = \frac{E_0''}{E_0} = \frac{n_i - n_j}{n_i + n_j}$$

Thus from the figure

$$E_0'' = E_0 R_{12} + r_{12} E_0 R_{23} r_{21} e^{i\phi} + r_{12} E_0 R_{23} R_{21} R_{23} r_{21} e^{i2\phi} + \ldots.$$

$$E_0'' = E_0 R_{12} + r_{12} E_0 R_{23} r_{21} e^{i\phi} \sum_{n=0}^{\infty} (R_{21} R_{23} e^{i\phi})^n$$

$$E_0'' = E_0 \left(R_{12} + \frac{r_{12} r_{21} R_{23}}{(e^{-i\phi} - R_{21} R_{23})} \right)$$

Similarly

$$E_0' = E_0 r_{12} r_{23} + E_0 r_{12} R_{23} R_{21} r_{23} e^{i\phi} + \ldots.$$

$$E_0' = E_0 \frac{r_{12} r_{23}}{1 - R_{21} R_{23} e^{i\phi}}$$

where the phase shift for the internally reflected wave is given by

$$\phi = \frac{2\pi(2d)}{\lambda_2} = \frac{\omega n_2 (2d)}{c}$$

Now for a plane wave

$$S_i = \frac{1}{2v_i} |E_{0i}|^2$$

Thus

$$R = \frac{S''}{S} = \frac{|E_0''|^2}{|E_0|^2}$$

$$T = \frac{v_1}{v_3} \frac{S'}{S} = \frac{n_3}{n_1} \frac{S'}{S}$$

From the above

$$R = \left[R_{12}^2 + \frac{2 r_{12} r_{21} R_{23} R_{12} (\cos\phi - R_{21} R_{23}) + (R_{12} r_{21} R_{23})^2}{\left(1 + (R_{21} R_{23})^2 - 2 R_{21} R_{23} \cos\phi\right)} \right]$$

$$T = \frac{n_3}{n_1} \frac{(r_{12} r_{23})^2}{\left(1 + (R_{21} R_{23})^2 - 2 R_{21} R_{23} \cos\phi\right)}$$

Since these two equations are simple functions of ϕ, which is linearly proportional to the frequency, they are simple functions of frequency which you should plot.

b) Since in part a) we used the convention that the incident wave is from the left, I will rephrase this question so that n_1 is are, n_2 is the coating, and n_3 is glass. In this case, we will have $n_1 < n_2 < n_3$, and $R_{21} R_{23} < 0$. Thus for T to be a maximum, from the above equation $\cos\phi = -1$, or $\phi = \pi$.

$$\phi = \frac{2\pi(2d)}{\lambda_2} = \pi \rightarrow d = \frac{\lambda_2}{4}$$

where λ_2 is the wavelength in the medium $= \frac{\lambda_1}{n_2}$