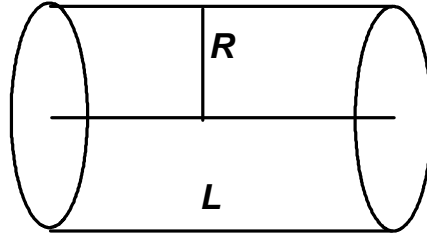


PHY 5347  
Homework Set 2 Solutions – Kimel

1. 8.5 a)



For the TM modes, we saw in class the resonance frequencies are  
TM:

$$\omega_{mnp} = \frac{1}{\sqrt{\epsilon\mu}} \sqrt{\frac{x_{mn}^2}{R^2} + \frac{p^2\pi^2}{L^2}} \quad \begin{array}{l} p = 0, 1, 2, \dots \\ m = 0, 1, 2, \dots \\ n = 1, 2, 3, \dots \end{array}$$

TE:

$$\omega'_{mnp} = \frac{1}{\sqrt{\epsilon\mu}} \sqrt{\frac{x_{mn}'^2}{R^2} + \frac{p^2\pi^2}{L^2}} \quad \begin{array}{l} p = 1, 2, 3, \dots \\ m = 0, 1, 2, \dots \\ n = 1, 2, 3, \dots \end{array}$$

Thus—

$$\frac{\omega_{mnp}}{\frac{1}{\sqrt{\epsilon\mu}}} = \sqrt{\frac{x_{mn}^2}{R^2} + \frac{p^2\pi^2}{L^2}}$$

$$\frac{\omega'_{mnp}}{\frac{1}{\sqrt{\epsilon\mu}}} = \sqrt{\frac{x_{mn}'^2}{R^2} + \frac{p^2\pi^2}{L^2}}$$

The lowest four frequencies are (in these units)

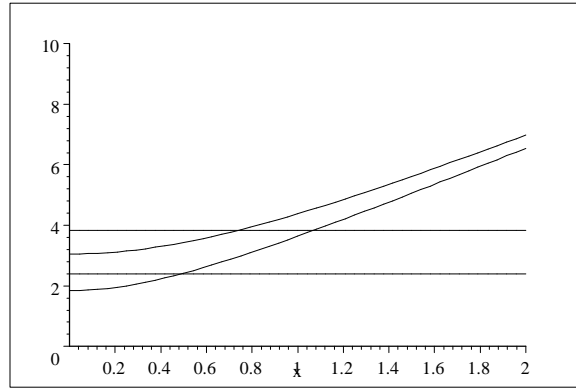
$$\omega_{010} = 2.405$$

$$\omega_{110} = 3.832$$

$$\omega'_{111} = \sqrt{1.841^2 + \pi^2 \left(\frac{R}{L}\right)^2}$$

$$\omega'_{211} = \sqrt{3.054^2 + \pi^2 \left(\frac{R}{L}\right)^2}$$

$$2.405, 3.832, \sqrt{1.841^2 + \pi^2 x^2}, \sqrt{3.054^2 + \pi^2 x^2}$$



where  $x = R/L$ .

The answer is "No."  $\omega'_{111}$  and  $\omega_{010}$  cross when

$$\sqrt{1.841^2 + \pi^2 x^2} = 2.405$$

or  $x = 0.49258$ . For frequencies smaller than this cross over frequency,  $\omega'_{111}$  is lowest, whereas for larger frequencies,  $\omega_{010}$  is lowest.