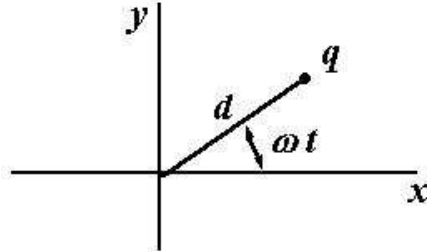


PHY 5347  
Homework Set 3 Solutions – Kimel

3.9.1 a)



$$\rho(\vec{x}, t) = q\delta(z)\delta(y - \sin\omega_0 t)\delta(x - d\cos\omega t)$$

To illustrate the equivalence of the two methods, I'll consider the lowest two moments.

$$n = 0 : Q(t) = \int \rho(\vec{x}, t) d^3x = q = \text{Re}(qe^{-i0\cdot\omega t})$$

$$n = 1 : \vec{p}(t) = \int \rho(\vec{x}, t) \vec{x} d^3x = qd(\hat{i}\cos\omega t + \hat{j}\sin\omega t) = \text{Re}[qd(\hat{i} + i\hat{j})e^{-i1\cdot\omega t}]$$

So we identify  $\vec{p} = qd(\hat{i} + i\hat{j})$  as the quantity to be used in Jackson's formulas.

Arbitrary n: The n'th multipoles will contribute with maximum frequencies of  $\omega_n = n\omega$ .

b) The proof that we can write

$$\rho(\vec{x}, t) = \rho_0(\vec{x}) + \sum_{n=1}^{\infty} \text{Re}[2\rho_n(\vec{x})e^{-in\omega t}]$$

with

$$\rho_n(\vec{x}) = \frac{1}{\tau} \int_0^{\tau} \rho(\vec{x}, t) e^{in\omega t} dt$$

was presented in lecture and will not be repeated here.

c) We have already calculated the  $n = 0, 1$  moments by the method of part a). Now we compute these moments by the method of part b).

$n = 0 :$

$$\rho_0(\vec{x}) = \frac{1}{\tau} \int_0^{\tau} [q\delta(z)\delta(y - \sin\omega_0 t)\delta(x - d\cos\omega t)] dt$$

$$Q = \int \rho_0(\vec{x}) d^3x = \frac{q}{\tau} \int_0^{\tau} dt \int d^3x [\delta(z)\delta(y - \sin\omega_0 t)\delta(x - d\cos\omega t)] = q$$

$n = 1 :$

$$\rho_1(\vec{x}) = \frac{1}{\tau} \int_0^\tau [q\delta(z)\delta(y - \sin \omega_0 t)\delta(x - d \cos \omega t)] e^{i\omega t} dt$$

$$\begin{aligned} \vec{p}(\vec{x}) &= \int d^3x \vec{x} (2\rho_1(\vec{x})) = \frac{2q}{\tau} \int_0^\tau dt \int d^3x \vec{x} [\delta(z)\delta(y - \sin \omega_0 t)\delta(x - d \cos \omega t)] \\ &= \frac{2qd}{\tau} \int_0^\tau dt e^{i\omega t} (\hat{i} \cos \omega t) + \sin \omega t = qd(\hat{i} + i \ ) \end{aligned}$$

as before.