Jackson Problem 2.7
B. J. Mattson

# 1 Part a

This part is easy (we did this part in class):

$$G(\vec{x},\vec{x'}) = [(x-x')^2+(y-y')^2+(z-z')^2]^{-\frac{1}{2}} - [(x-x')^2+(y-y')^2+(z+z')^2]^{-\frac{1}{2}} \quad (1)$$

# 2 Part b

To do this, I need:

$$\begin{aligned} \nabla G(\vec{x},\vec{x'})\cdot\hat{n} &= -\frac{\partial}{\partial z'}G_D(\vec{x},\vec{x'})|_{surface} && (2) \\ &= -\frac{2z}{[(x-x')^2+(y-y')^2+z^2]^{\frac{3}{2}}} && \\ & && (3) \end{aligned}$$

Converting to cylindrical coordinates: $x=\rho\cos\phi$, $y=\rho\sin\phi$:

$$\begin{aligned} (x-x')^2+(y-y')^2+z^2 &= (\rho\cos\phi-\rho'\cos\phi')^2+(\rho\sin\phi-\rho'\sin\phi')^2+z^2 && (4) \\ &= \rho^2+\rho'^2-2\rho\rho'\cos(\phi-\phi')+z^2 && (5) \\ & && (6) \end{aligned}$$

So,

$$\nabla' G(\vec{x},\vec{x'})\cdot\hat{n} = \frac{-2z}{(\rho^2+\rho'^2-2\rho\rho'\cos(\phi-\phi')+z^2)^{\frac{3}{2}}} \quad (7)$$

And the integral expression for the potential anywhere is:

$$\Phi(\rho,\phi,z) = \frac{1}{4\pi}\int_0^{2\pi} d\phi' \int_0^a \rho'\, d\rho' \frac{2z\,V}{(\rho^2+\rho'^2-2\rho\rho'\cos(\phi-\phi')+z^2)^{\frac{3}{2}}} \quad (8)$$

## 3 Part c

So, just evaluate $\Phi(0, \phi, z)$:

$$\Phi(0,\phi,z) = \frac{Vz}{4\pi}\int_0^{2\pi} d\phi' \int_0^a \rho'\, d\rho' \frac{2\,V}{(\rho'^2+z^2)^{\frac{3}{2}}} \tag{9}$$

$$= \frac{Vz(2\pi)}{4\pi}\left[-2(\rho'^2+z'^2)^{-\frac{1}{2}}\right]_0^a \tag{10}$$

$$\Phi(0,\phi,z) = V\left(1-\frac{1}{\sqrt{a^2+z^2}}\right) \tag{11}$$

$$\tag{12}$$