Jackson Problem 5.6
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First consider a cylinder with no hole bored through it. Then use Ampere's law to find the magnetic field.

$$\oint \vec{B}_c \cdot d\vec{l} = \mu_0 I_{encl} \tag{1}$$
$$B_c \cdot s\pi r = \mu_0 J(\pi r^2) \tag{2}$$
$$\vec{B}_c = \frac{\mu_0 J}{2} r\hat{\phi} \tag{3}$$

Now suppose that we have a cylinder with a current density of $-J$ running just through a hole of radius b centered a distance d from the cylinder's axis. Also take a coordinate system centered on the center of that hole (and figure the conversion out later). The magnetic field in the hole is then

$$\vec{B}_h = -\frac{\mu_0 J}{2} r_h \hat{\phi}_h \tag{4}$$

In order to superpose $\vec{B}_c$ and $\vec{B}_h$, I need to relate the two coordinate systems. After a bit of geometry, I find that (with the x-axis of the original coordinate system passing through the center of the hole)

$$r_h = \sqrt{(x-d)^2 + y^2} \tag{5}$$
$$\hat{\phi}_h = \frac{-y\hat{x} + (x-d)\hat{y}}{\sqrt{(x-d)^2 + y^2}} \tag{6}$$

Then the total magnetic field for the original problem is

$$\vec{B} = \vec{B}_c + \vec{B}_h \tag{7}$$
$$= \frac{\mu_0 J}{x}\left[\sqrt{(x-d)^2 + y^2}\left(\frac{-y\hat{x} + (x-d)\hat{y}}{\sqrt{(x-d)^2 + y^2}}\right) + \sqrt{x^2 + y^2}\left(\frac{-y\hat{x} + x\hat{y}}{\sqrt{x^2 + y^2}}\right)\right] \tag{8}$$
$$= \frac{\mu_0 J d}{2}\hat{y} \tag{9}$$