Jackson Problem 9.3
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We saw in chapter 3 (or there abouts) that this configuration "far away" basically has the field of a dipole. From Jackson's equation (3.36) for a static problem with the same geometry, the dipole term of the potential is:

$$\Phi(r,\theta) = V\left[\frac{3}{2}\left(\frac{R}{r}\right)^2 P_1(\cos\theta)\right] \tag{1}$$

For us, this would be:

$$\Phi(r,\theta) = Ve^{-i\omega t}\left(\frac{3}{2}\frac{R^2}{r^2}\cos\theta\right) \tag{2}$$

But, in general, the potential due to a dipole is:

$$\Phi(r,\theta) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{x}}{r^3} = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2} \tag{3}$$

Comparing these,

$$\vec{p} = \left(6\pi\epsilon_0 V e^{-i\omega t} R^2\right)\hat{z} \tag{4}$$

Then,

$$\begin{aligned}
\vec{H} &= \frac{ck^2}{4\pi}(\hat{n}x\vec{p})\frac{e^{ikr}}{r} && (5)\\
&= \frac{ck^2}{4\pi}(\hat{r}x\vec{p})\frac{e^{ikr}}{r} && (6)\\
&= \frac{ck^2}{4\pi}\left(6\pi\epsilon_0 R^2 V e^{-i\omega t}\right)\frac{e^{ikr}}{r}(\hat{r}x[\hat{r}\cos\theta - \hat{\theta}\sin\theta]) && (7)\\
&= -\frac{3}{2}\frac{R^2V}{z_0}\frac{e^{ikr}}{r}e^{-i\omega t}\sin\theta\hat{\phi} && (8)
\end{aligned}$$

And,

$$\begin{aligned}
\vec{E} &= z_0(\vec{H}x\hat{n}) && (9)\\
\vec{E} &= -\frac{3}{2}R^2V\frac{e^{ikr}}{r}e^{-i\omega t}\sin\theta\hat{\theta} && (10)
\end{aligned}$$

Also

$$\frac{dP}{d\Omega} = \frac{z_0 c^2 k^4}{32\pi^2} |p|^2 \sin^2\theta \tag{11}$$

$$= \frac{z_0 c^2 k^4}{32\pi^2} \left(36\pi^2 \epsilon_0^2 R^4 V^2\right) \sin^2\theta \tag{12}$$

$$\frac{dP}{d\Omega} = \frac{9}{8} \frac{k^4}{z_0} R^4 V^2 \sin\theta \tag{13}$$

Finally, the total power is:

$$P = \frac{c^2 z_0 k^4}{12\pi} |p^2| \tag{14}$$

$$P = 3\pi \frac{k^4}{z_0} R^4 V^2 \tag{15}$$