

We already know that

$$u_{\parallel} = \frac{u'_{\parallel} + v}{1 + \vec{v} \cdot \vec{u}'} \quad (1)$$

$$\vec{u}_{\perp} = \frac{\vec{u}'_{\perp}}{\gamma_v (1 + \vec{v} \cdot \vec{u}')} \quad (2)$$

where the boost is in the x direction. Also,

$$t = t' \gamma_v + v \gamma_v x' = \gamma_v (t' + v x') \quad (3)$$

So,

$$dt = \gamma_v (dt' + v dx') \quad (4)$$

Then we can find a_{\parallel} by

$$a_{\parallel} = \frac{du_{\parallel}}{dt} \quad (5)$$

$$= \frac{d\left(\frac{u'_{\parallel} + v}{(1 + \vec{v} \cdot \vec{u}')}\right)}{dt} \quad (6)$$

$$= \frac{d\left(\frac{u'_{\parallel} + v}{(1 + \vec{v} \cdot \vec{u}')}\right)}{\gamma_v (dt' + v dx')} \quad (7)$$

$$= \frac{(1 + \vec{v} \cdot \vec{u}') du'_{\parallel} - (u'_{\parallel} + v) (\vec{v} \cdot d\vec{u}')}{(1 + \vec{v} \cdot \vec{u}')^2 \gamma_v (dt' + v dx')} \quad (8)$$

$$= \frac{(1 + \vec{v} \cdot \vec{u}') \frac{du'_{\parallel}}{dt'} - (u'_{\parallel} + v) \left(\vec{v} \cdot \frac{d\vec{u}'}{dt'}\right)}{\gamma_v (1 + \vec{v} \cdot \vec{u}')^2 \left(1 + v \frac{dx'}{dt'}\right)} \quad (9)$$

$$= \frac{(1 + \vec{v} \cdot \vec{u}') a'_{\parallel} - (u'_{\parallel} + v) v a'_{\parallel}}{\gamma_v (1 + \vec{v} \cdot \vec{u}')^2 (1 + \vec{v} \cdot \vec{u}')} \quad (10)$$

$$= \frac{a'_{\parallel} (1 + v u'_{\parallel} - u'_{\parallel} - v^2) (1 - v^2)^{\frac{1}{2}}}{(1 + \vec{v} \cdot \vec{u}')^3} \quad (11)$$

$$\vec{a}_{\parallel} = \frac{\vec{a}'_{\parallel} (1 - v^2)^{\frac{3}{2}}}{(1 + \vec{v} \cdot \vec{u}')^3} \quad (12)$$

Then, for a_{\perp} ,

$$\vec{a}_{\perp} = \frac{d \frac{\vec{u}'_{\perp}}{\gamma_v(1+\vec{v}\cdot\vec{u}')}}{dt} \quad (13)$$

$$= \frac{(1+\vec{v}\cdot\vec{u}') d\vec{u}'_{\perp} - \vec{u}'_{\perp}(\vec{v}\cdot d\vec{u}')}{\gamma_v(1+\vec{v}\cdot\vec{u}')^2 \gamma_v(dt'+v dx')} \quad (14)$$

$$= \frac{(1+\vec{v}\cdot\vec{u}') \vec{a}'_{\perp} - \vec{u}'_{\perp}(\vec{v}\cdot\vec{a}')}{\gamma_v^2(1+\vec{v}\cdot\vec{u}')^3} \quad (15)$$

$$= \frac{(1-v^2) [\vec{a}'_{\perp} + (\vec{v}\cdot\vec{u}') \vec{a}'_{\perp} - \vec{u}'_{\perp}(\vec{v}\cdot\vec{a}')] }{(1+\vec{v}\cdot\vec{u}')^3} \quad (16)$$

$$\vec{a}_{\perp} = \frac{1-v^2}{(1+\vec{v}\cdot\vec{u}')^3} [\vec{a}'_{\perp} + \vec{v} \times (\vec{a}' \times \vec{u}')] \quad (17)$$