

1 Part a

In any frame,

$$P_\mu P^\mu = M^2 \quad (1)$$

$$p_{1\mu} p_1^\mu = m_1^2 \quad (2)$$

$$p_{2\mu} p_2^\mu = m_2^2 \quad (3)$$

And from conservation of energy and momentum,

$$p_{2\mu} = P_\mu - p_{1\mu} \quad (4)$$

So,

$$p_{2\mu} p_2^\mu = (P_\mu - p_{1\mu})(P^\mu - p_1^\mu) \quad (5)$$

$$= P_\mu P^\mu - 2P_\mu p_1^\mu + p_{1\mu} p_1^\mu \quad (6)$$

Or

$$m_2^2 = M^2 + m_1^2 - 2P_\mu p_1^\mu \quad (7)$$

Now, in the rest frame of the decaying particle,

$$P_\mu = \begin{pmatrix} M \\ \vec{0} \end{pmatrix} \quad (8)$$

$$p_1^\mu = \begin{pmatrix} E_1 \\ -\vec{p}_2 \end{pmatrix} \quad (9)$$

So $P_\mu p_1^\mu = E_1 M$ in any frame. Then $m_2^2 = M^2 + m_1^2 - 2ME_1$. Or,

$$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M} \quad (10)$$

Finding E_2 is pretty much the same,

$$E_2 = \frac{M^2 + m_2^2 - m_1^2}{2M} \quad (11)$$

2 Part b

In the decaying particle's rest frame $\vec{p}_1 = -\vec{p}_2$. From Jackson's equation (11.55),

$$E_i^2 = m_i'^2 = p_i^2 + m_i^2 \quad (12)$$

$$T_i = m_i' - m_i \quad (13)$$

So

$$(T_i + m_i)^2 = p_i^2 + m_i^2 \quad (14)$$

$$T_i^2 + 2T_i m_i + m_i^2 = p_i^2 + m_i^2 \quad (15)$$

$$p_i^2 = T_i^2 + 2T_i m_i \quad (16)$$

Now $\vec{p}_i = -\vec{p}_j$ ($i = 1$ or 2 , $j = 2$ or 1) So,

$$|\vec{p}_i| = |-\vec{p}_j| \quad (17)$$

Or,

$$T_i^2 + 2m_i T_i = T_j^2 + 2m_j T_j \quad (18)$$

Also $\Delta M = M - m_1 - m_2 = T_1 + T_2$. So,

$$T_i = \Delta M - T_j \quad (19)$$

Then, combining equations (16), (18), and (19),

$$T_i^2 + 2m_i T_i = (\Delta M - T_i)^2 + 2m_j (\Delta M - T_i) \quad (20)$$

$$T_i^2 + 2m_i T_i = (\Delta M)^2 - T_i \Delta M + T_i^2 + 2m_j \Delta M - 2T_i m_j \quad (21)$$

Solving for T_i ,

$$T_i = \frac{(\Delta M)^2 + 2m_j \Delta M}{2m_i + 2m_j + 2\Delta M} \quad (22)$$

$$= \frac{(\Delta M)^2 + 2m_j \Delta M}{2\Delta M \left(\frac{m_i + m_j}{\Delta M} + 1 \right)} \quad (23)$$

$$= \frac{\Delta M + 2(M - m_i - \Delta M)}{2 \left(\frac{M - \Delta M}{\Delta M} + 1 \right)} \quad (24)$$

$$= \frac{\Delta M}{2} \left(\frac{\Delta M + 2M - 2m_i - 2\Delta M}{M - \Delta M + \Delta M} \right) \quad (25)$$

$$= \Delta M \frac{1}{2M} (2M - 2m_i - \Delta M) \quad (26)$$

So, finally,

$$T_i = \Delta M \left(1 - \frac{m_i}{M} - \frac{\Delta M}{2M} \right) \quad (27)$$

3 Part c

This is just plug and chug....

$$T_\mu = 4.116 \text{ MeV} \quad (28)$$

$$T_\nu = 29.784 \text{ MeV} \quad (29)$$